The Fundamental Studies on the Dissipation of the Flow at the Straight Drop

Katsurō MAEKAWA
(Laboratory of Irrigation and Drainage, Faculty of Agriculture, Yamagata University)
Received Aug. 31 st, 1968

Introduction

The drop structure is one of the aqueduct structures which lead from irrigation source to irrigation water. The rapid flow structure is contained within the drop structure in a broader sense.

The drop structure changes the stream in a waterway into the flow of the drop and elevation head into velocity head, that is to say, causes it to erode and destroy. Accordingly, it needs to preparate the stilling basin of reasonable structure in this point and dissipate.

A foundation of design for the drop structure which has been constructed, consists in the following basic assumption. That is, the back-pressure of the nappe maintains atmospheric pressure. On the basis of it, the length of the stilling basin has been determined in consideration of the horizontal distance which the nappe reached from the drop wall. In case the length does not suit, it can not give full play to its function.

On the contrary the satisfying design of the stilling basin needs the much cost of the construction. On the opposite consideration of the former case there, this study was performed in the state that the back of the nappe is the negative pressure and the non-ventilation. It is the purpose of this experiment to diminish the energy of the nappe, make the horizontal distance (which the nappe reaches) short and decrease specially the length of the stilling basin in order to make the rushing angle of the nappe large, and look forward to safety of the waterway.

The writer assumed the negative pressure -the pressure behind the nappe- is smaller than atmospheric pressure. In this case the observed position was indicated in Fig. 2, 3.

Non-ventilation means the state that the back of the nappe is filled with water. In the case of the low drop, if the both sides of the nappe touched on the chanal wall, that is, if the back of the nappe was cut off the open air, it results the state of non-ventilation.

Notations

h_c =the critical depth= $\sqrt[3]{\frac{q^2}{g}}$

L_c =the distance from the drop wall (A-A section) to the critical depth.

h =height of the drop.

h_o =the depth of flow at the crest of the drop.

h_p =the depth of the under-nappe pool between the drop wall and the nappe.

P_2 =pressure of an air between the drop wall and the nappe (cm).

P_1 =atmospheric pressure (cm).

L =the horizontal distance which the nappe reaches from the drop wall, only the back of the nappe is the non-ventilative state.

L_x =the horizontal distance which the nappe reaches from the drop wall, only the back of the nappe is the atmospheric pressure.

L_x' =the horizontal distance which the nappe reaches from the drop wall, only the back of the nappe is the negative pressure.

θ =rushing angle of the nappe.

h_1 =the depth at the toe of the nappe.

h_2 =the depth in the downstream (position 2 m from the drop wall).

H =the depth in the upstream$=\frac{3}{2}h_c$

D_n =drop number $=\frac{q^2}{gh^3}=\left(\frac{h_c}{h}\right)^3$

q =the discharge per unit width.

g =acceleration of gravity.

L_J =the length of the hydraulic jump, only the back of the nappe is the negative or the atmospheric pressure.

L_E =the distance from the drop wall to the end of the hydraulic jump, only the back of the nappe is the negative or the atmospheric pressure.

L_B =the distance from the drop wall to the beginning of the hydraulic jump, only the back of the nappe is the negative or the atmospheric pressure.

L_J' =the length of the hydraulic jump, only the back of the nappe is the non-ventilative state.

L_E'=the distance from the drop wall to the end of the hydraulic jump, only the back of the nappe is the non-ventilative state.

L_B'=the distance from the drop wall to the beginning of the hydraulic jump, only the back of the nappe is the non-ventilative state.

F_o =Froude number at the crest of the drop.

F_2 =Froude number at the downstream. (position 2 m from the drop wall)

Equipment for Hydraulic Experiment

The experimental channel was a rectangular section (width 0.5 m, length 10 m and height 0.5 m of side wall), moreover the alterable drop was set up at the position (5 m) from the downstream end. Backwater was changed at the downstream end at distcretion. This experiment is the channel gradient (1/1000) with the up and down stream, the channel gradient changes to 1/20.

Glazing (2 m) both the up and down streams from the drop portion, a lateral (a) and a front figure (b) of the drop portion are indicated at Fig. 2.

By preparing the projecting part in the drop portion, the nappe reached speedily

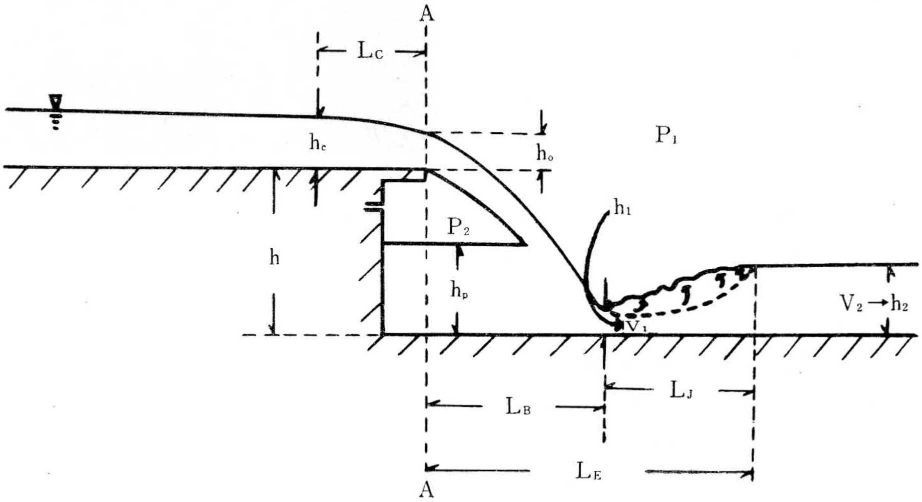

Fig. 1.

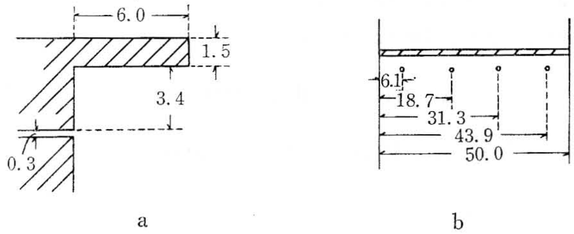

Fig. 2. (a) Lateral figure of drop portion (unit……cm)
(b) Front figure of drop portion (unit……cm)

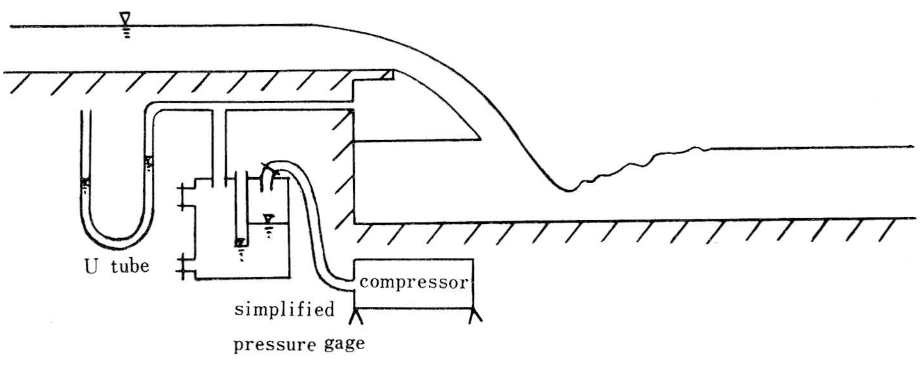

Fig. 3. Equipment for hydraulice experiment

the downstream channel. But the writer could not analyze effect that should have influence upon the nappe-shape and so on.

The relation of entrance and exit of air behind the under-nappe was indicated in Fig. 3. Perhaps the pressure behind the under-nappe should not naturally occur to the positive pressure, and yet in order to study the characteristic of the flow it was observed.

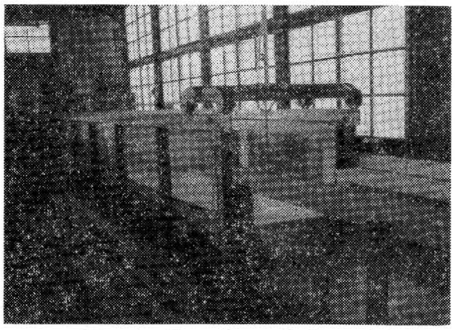

Fig. 4. Experimental channel

By the compressor in the case of the positive pressure, by the simplified pressure gage in the case of the negative pressure, the state was made and measured through a U tube. The experiment in the case of the alterable pressure behind the under-nappe was performed by the discharge of the four classes (15.967, 22.935, 26.306, 31.743…… unit l/s) at the height (22 cm) of the drop. The under-nappe shape was measured through the side wall on the glazing.

The non-ventilative state was performed by the four classes (15.967, 22.935, 26.306, 31.743……unit l/s) at the four classified height (22, 17, 12, 7……unit cm) of the drop. The writer assumed the beginning point of the wave flow that began when lowered the downstream elevation from the complete submerged flow, whose end point was kept when lowered the lower downstream elevation.

The experimental channel indicated in Fig. 4.

Results and Discussion

I A Position where the Critical Depth arises

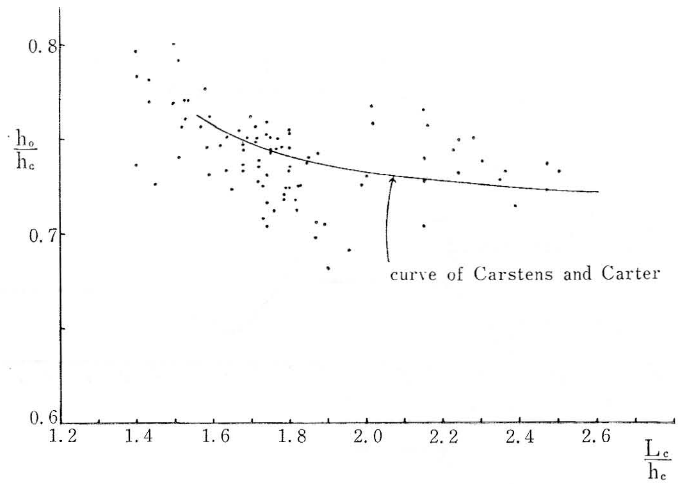

Fig. 5. Relationship of a position where the critical depth arises in the alterable pressure

It was said that the critical depth for the free over-flow at the drop arose on the crest of the drop, but if the backwater on the crest is not, the position is in the upper stream than the crest of the drop.

1) *positive and negative pressure*

Relationship of the position where the critical depth arises by the alterable pressure of the under-nappe, corresponded approximately with the figure of Carstens, M. R. and R. W. Carter. It is indicated in Fig. 5.

2) *non-ventilation*

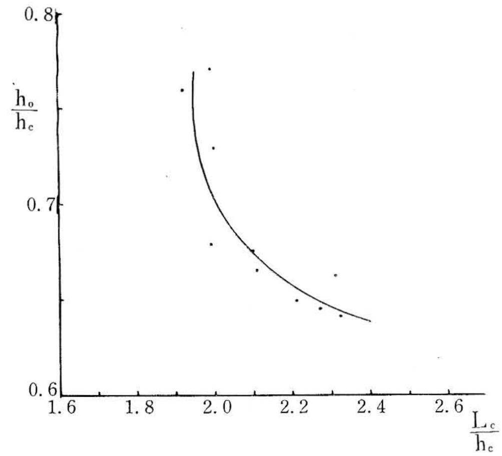

Fig. 6. Relationship of a position where critical depth arises in the non-ventilation

In cases the values of Fig. 6 cause the jet flow in the downstream, this is different from Fig. 5 and changed itself with the state of the downstream, such as the water cushion causes at the toe of the nappe.

II Drop Portion

1) *nappe-shape*

The nappe-shape equation in the alterable pressure behind the nappe was indicated apart from the positive and the negative pressure. The basic equation of it was indicated an expression (1). The experimental equations were indicated in Table (1) and their data were indicated in Fig. 7~11, only $-0.32 \leqq \frac{P_2-P_1}{H} \leqq 0.13$.

$$\frac{x}{H} = a\left(\frac{y}{H}+\frac{h_0}{H}\right)^b \cdots\cdots(1)$$

Table 1.

		positive pressur	negative pressure	atmospheric pressure	
				from positive pressure	from negative pressure
over-nappe	$\frac{h_0}{H}$	$0.126\left(\frac{P_2-P_1}{H}\right)+0.496$	$0.093\left(\frac{P_2-P_1}{H}\right)+0.496$	0.496	0.496
	a	$2.418\left(\frac{P_2-P_1}{H}\right)+1.519$	$1.297\left(\frac{P_2-P_1}{H}\right)+1.520$	1.519	1.520
	b	$0.455\left(\frac{P_2-P_1}{H}\right)+0.572$	$0.391\left(\frac{P_2-P_1}{H}\right)+0.578$	0.572	0.578
under-nappe	a	$3.805\left(\frac{P_2-P_1}{H}\right)+1.467$	$1.801\left(\frac{P_2-P_1}{H}\right)+1.470$	1.467	1.470
	b	$0.312\left(\frac{P_2-P_1}{H}\right)+0.563$	$0.271\left(\frac{P_2-P_1}{H}\right)+0.565$	0.563	0.565

x : x-axial co-ordinate of the nappe (plus on the lower course)

y : y-axial co-ordinate of the nappe (plus on the gravitational course)

In comparison with the positive and the negative pressure at the state $\frac{P_2-P_1}{H}=0$, the equation of the over and under nappe-shape from the positive pressure is approxi-

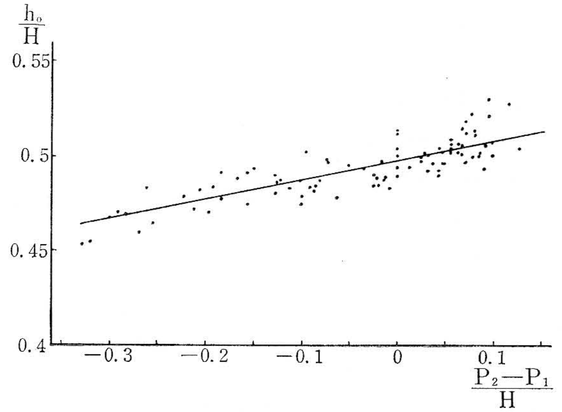

Fig. 7. Depth at the crest of the drop in the alterable pressure

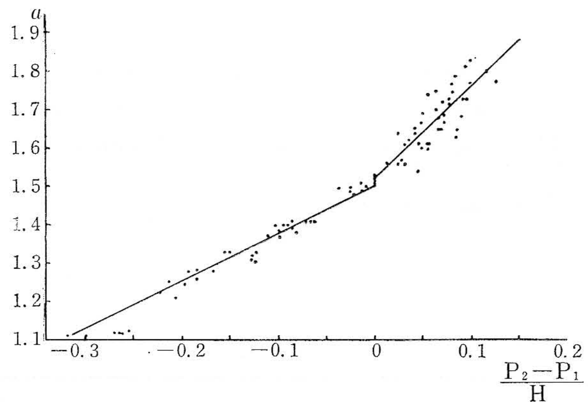

Fig. 8. Coefficient a of equation (1) in the alterable pressure for the over-nappe

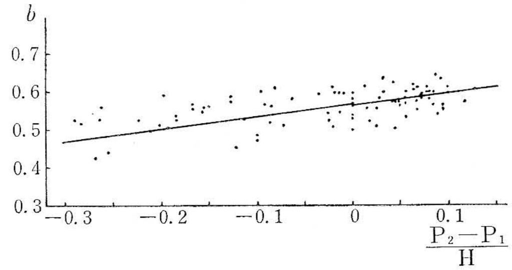

Fig. 9. Coefficient b of equation (1) in the alterable pressure for the over-nappe

mately equal to it of the negative pressure. Accordingly if the back-pressure was equal to the atmospheric pressure, the equation is as follows.

over-nappe shape

$$\frac{x}{H} = 1.520\left(\frac{y}{H} + 0.496\right)^{0.575} \quad \cdots\cdots (2)$$

under-nappe shape

$$\frac{x}{H} = 1.496\left(\frac{y}{H}\right)^{0.564} \quad \cdots\cdots (3)$$

Namely, the depth h_o of flow at the crest of the drop

$$\frac{h_0}{H} = 0.496$$

$$\therefore\ h_0 = 0.496\,H = 0.744\,h_C \quad \cdots\cdots (4)$$

since $H = 1.5\,h_C$

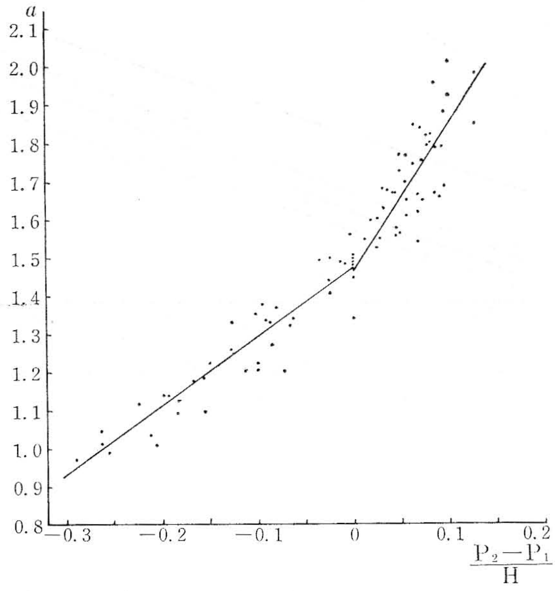

Fig. 10. Coefficient a of equation (1) in the alterable pressure for the under-nappe

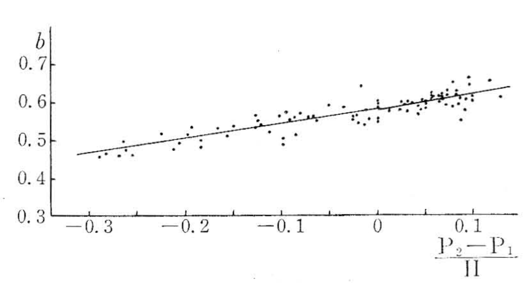

Fig. 11. Coefficient b of equation (1) in the alterable pressure for the under-nappe

Each of the scientists of Dr. H. Rouse[1], Dr. T. Iwasaki[2], Dr. T. Kikuoka[3], Dr. T. Yamamoto[4], Master. K. Izutsu[5] and others studied the flow of the drop in the case that the back-pressure of the under-nappe is equal to the atmospheric pressure, only expect the rapid flow. The experimental equation of the nappe-shape by the writer is on the safe side when compared that much, and the depth at the crest of the drop is roughly from four percent to twenty percent.

The depth h_0 in the non-ventilative state was experimentally indicated by the following equation, only in the case that the jet flow causes in the downstream.

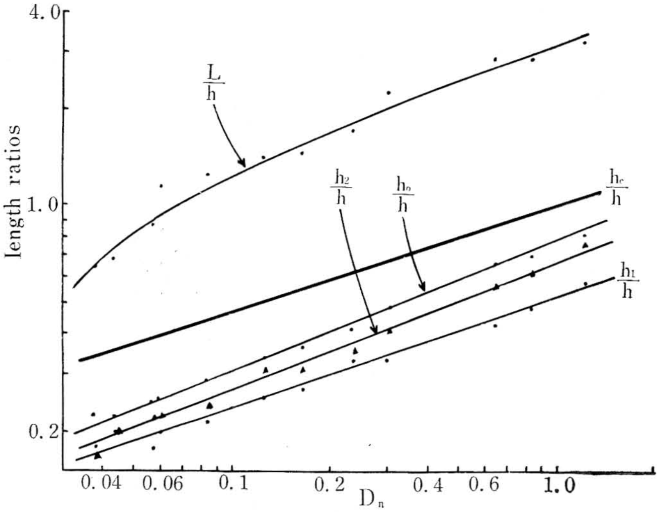

Fig. 12. Flow geometry at the straight drop in the case that jet flow causes in the downstream in the non-ventilation

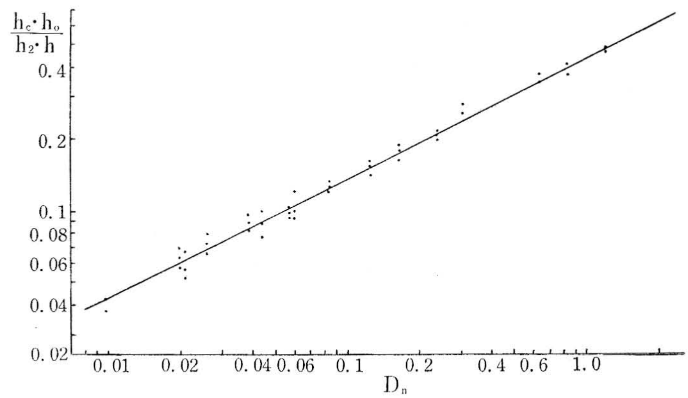

Fig. 13. Relation between $\frac{h_0}{h_2}$ and Dn in the case that water cushion cosists in the toe at the non-ventilation

$$h_0=0.780\ h \cdot \left(\frac{h_c}{h}\right)^{1.192}=0.780\ h \cdot D_n^{0.397} \quad \cdots\cdots(5)$$

In the case that the water cushion consists in the toe

$$h_0=0.441\ h_2 \cdot \left(\frac{h_c}{h}\right)^{0.516}=0.441\ h_2 \cdot D_n^{0.172} \quad \cdots\cdots(6)$$

in this case, coefficient of the basic equation (1) was indicated in Fig. 13～15.

The study for the non-ventilative state had been performed by Dr. K. Ōtsubo[6] and Dr. K. Ashida[7], and it must be increasingly neccessary to study in the future though the flow of the low drop, becomes an issue at present in connection with the wave flow.

2) *coefficient of discharge*

Coefficient of the discharge for the flow of the drop is indicated by the following equation.

$$m=\frac{Q}{\sqrt{2g}\cdot B\cdot h_0^{\frac{2}{3}}}=\frac{1}{\sqrt{2}}\left(\frac{h_0}{h_c}\right)^{-\frac{3}{2}} \quad \cdots\cdots(7)$$

m : coefficient of discharge

Q : discharge (m^3/s)

B : width of the channel (m)

It changes with the alterable pressure of the under-nappe.

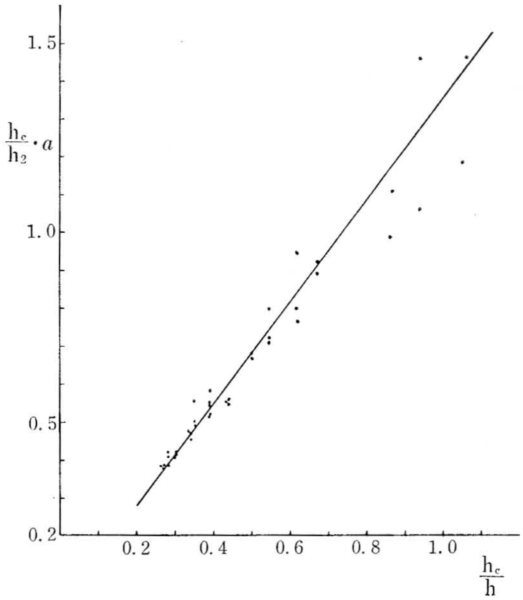

Fig. 14. Coefficient *a* of equation (1) in the case that water cushion cosists in the toe at the non-ventilation

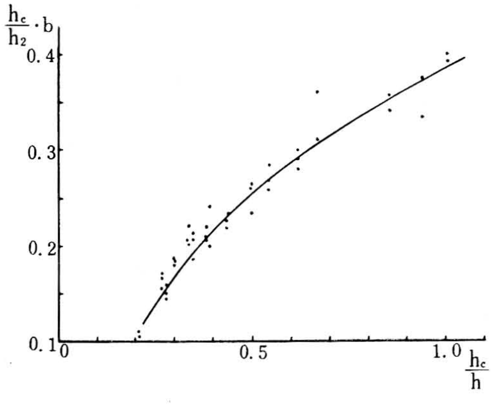

Fig. 15. Coefficient *b* of equation (1) in the case that water cushion consists in the toe at the non-ventilation

positive pressure

$$m=\frac{1}{\sqrt{2}}\left\{0.189\left(\frac{P_2-P_1}{H}\right)+0.744\right\}^{-\frac{3}{2}} \quad \cdots\cdots (8)$$

negative pressure

$$m=\frac{1}{\sqrt{2}}\left\{0.140\left(\frac{P_2-P_1}{H}\right)+0.744\right\}^{-\frac{3}{2}} \quad \cdots\cdots (9)$$

In the case that the back-presssure of the under-nappe is equal to the atmospheric pressure

$$m=\frac{1}{\sqrt{2}}(0.744)^{-\frac{3}{2}} \fallingdotseq 1.10 \quad \cdots\cdots (10)$$

3) *angle of inrush*

Differentiating the equation (1)

$$\frac{dy}{dx}=\frac{1}{a^{\frac{1}{b}}\cdot b}\left(\frac{x}{H}\right)^{\frac{1}{b}-1}=\tan\theta \quad \cdots\cdots (11)$$

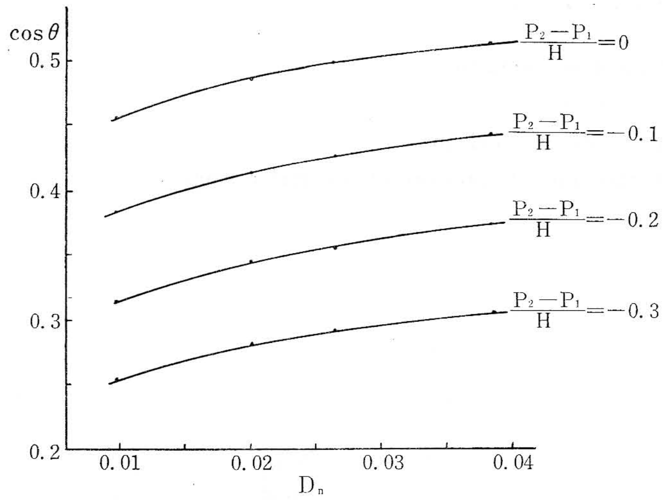

Fig. 16. Change of the angle of inrush with the alterable prersure

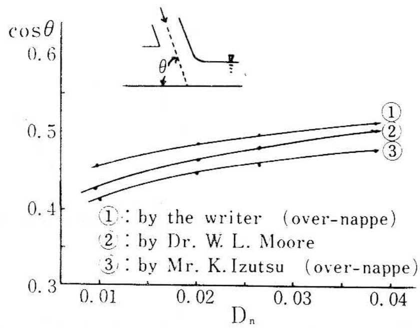

Fig. 17. Angle of inrush in the case that the back of the nappe is the atmospheric pressure

As the values of a and b change with the back-pressure of the nappe, the relationship between D_n and cos θ was indicated in Fig. 16, in experimental limits. If it was the negative pressure, the angle of inrush will be naturally large in comparison with the atmospheric pressure. In the case that it is equal to atmospheric pressure, the angle of inrush in disregard of energy loss is as follows.

$$\cos\theta = \frac{1.06}{\sqrt{\frac{h}{h_c} + \frac{3}{2}}} \quad \cdots\cdots (12)$$

It was indicated in Fig. 17 in comparison with the theorical equation of W. P. White.

4) *the horizontal distance which the nappe reaches*

The horizontal distance which the nappe reaches, can be computed at the basic epuation (1) from the known H and y through the experimental equation. As it changed with the alterable pressure of the back, its relation was indicated in Fig. 18, and the proportion of the decrease was indicated in Fig. 19 in comparison with the atmospheric pressure. It decreases in thirty percent at $\frac{P_2 - P_1}{H} = -0.3$, in experimental limits.

Dr. T. Naitō, Dr. T. Yamamoto reached theorically $h_0 = 0.656\ h_c$ and indicated the following equation at the over-nappe.

$$x = \frac{q}{0.656\,h_c}\sqrt{\frac{(2h+h_0)}{g}} \quad \cdots\cdots (13)$$

That of the writer is compared in Fig. 20.

5) *the depth h_p of the under-nappe*

The depth of the under-nappe pool between the drop and the nappe plays the

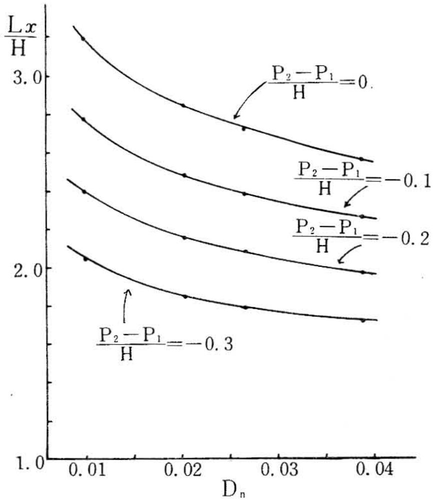

Fig. 18. Horizontal distance which the nappe reaches, at the negative pressure or the atmospheric pressre

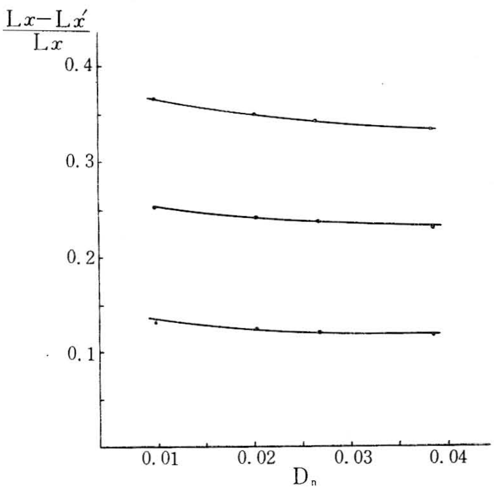

Fig. 19. Proportion of horizontal distance which the napp reaches

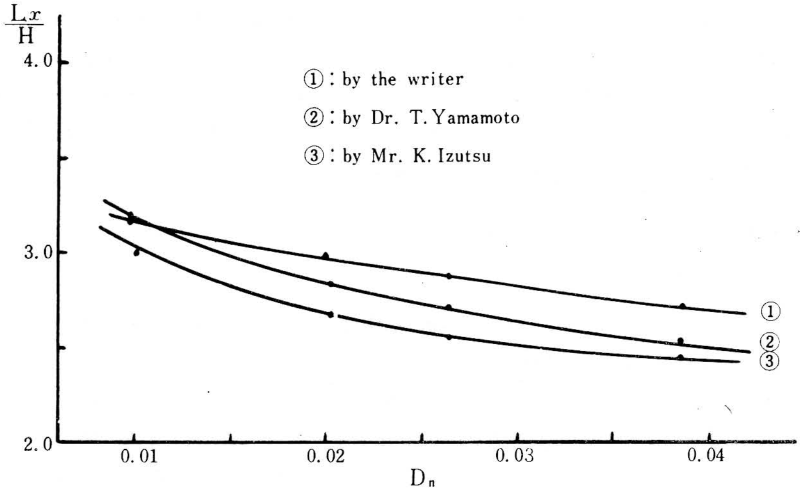

Fig. 20. Comparison with the horizontal distance which the nappe reaches

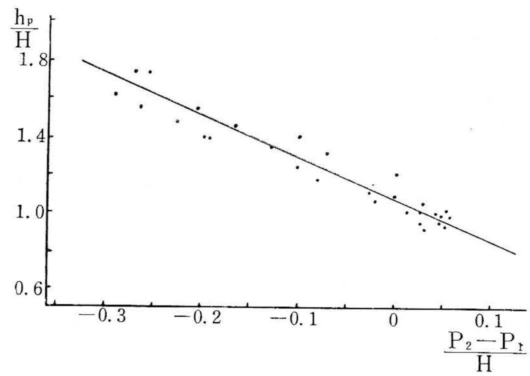

Fig. 21. h_p by the alterable pressure in the case that jet flow occurs in the downstream

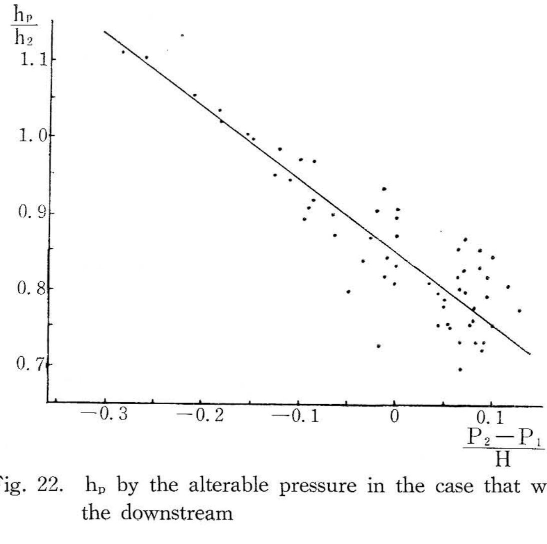

Fig. 22. h_p by the alterable pressure in the case that water cushion occurs in the downstream

important part on the dissipation. Change of h_p with the alterable pressure behind the nappe was carried on apart the jet flow and the water cushion in the downstream, and was indicated in Fig. 21, 22.

The equation is experimentally as follows.

$$\left.\begin{array}{l} \text{jet flow} \\ \quad \dfrac{h_p}{H} = 2.413\left(\dfrac{P_2-P_1}{H}\right)+1.08 \\ \text{water cushion} \\ \quad \dfrac{h_p}{h_2} = 0.975\left(\dfrac{P_2-P_1}{H}\right)+0.85 \end{array}\right\} \cdots\cdots \quad (14)$$

$$\left.\begin{array}{l} \text{jet flow} \\ \quad h_p=1.08\ \mathrm{H}=1.62\ h_c \\ \text{water cushion} \\ \quad h_p=0.85\ h_2 \\ \\ ,\ \text{at}\ \dfrac{P_2-P_1}{H}=0 \end{array}\right\} \cdots\cdots \quad (15)$$

The experimental equation by the writer is compared with the experimental equation of W. Rand[9] and the theorical that of W. L. Moore[10] in Fig. 23.

As both of the equation have been considered by the depth at the toe, that by the writer is necessary to be connected with the case that the jet flow or the water cushion causes in the downstream. It puts still a problem.

III Hydraulic Jump

As the point A in Fig. 24 would consist of which the velocity is zero on the surface of the flow, it was assumed the end of hydraulic jump. This method could be readily observed in comporison with that at the arising point of the bubble. The relationship between the alterable pressure of the back and the length of the hydraulic jump was indicated in Fig. 25~27 and by the following equation.

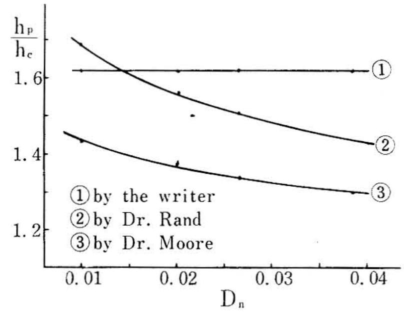

Fig. 23. Comparison with the depth h_p

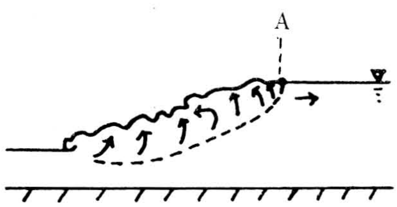

Fig. 24.

$$\left.\begin{aligned} L_J &= 4\ h_2 \\ \frac{L_E}{h_2} &= 1.57\left(\frac{P_2-P_1}{H}\right)+5.3 \\ \frac{L_B}{h_2} &= 1.00\left(\frac{P_2-P_1}{H}\right)+1.25 \end{aligned}\right\} \cdots\cdots (16)$$

$$\left.\begin{aligned} L_J &= 4\ h_2 \\ L_E &= 5.3\ h_2 \\ L_B &= 1.25\ h_2 \\ &\text{since} \\ \frac{P_2-P_1}{H} &= 0 \end{aligned}\right\} \cdots\cdots (17)$$

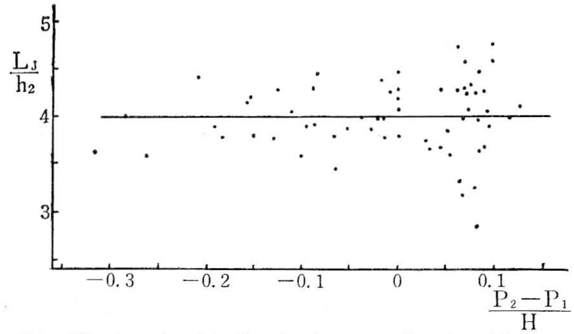

Fig. 25. The length of hydraulic jump in the alterable pressure

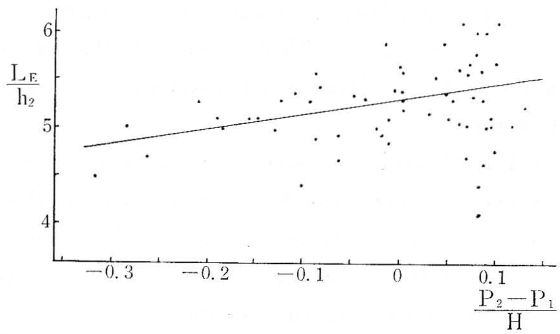

Fig. 26. The distance from the drop wall to the end of hydraulic jump in the alterable pressure

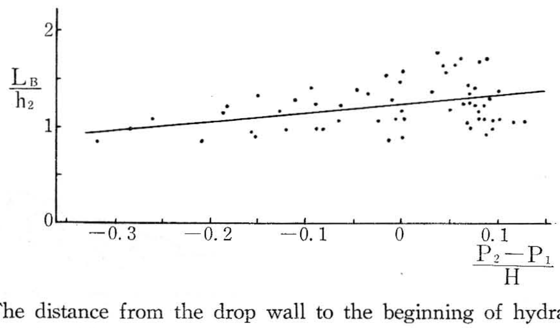

Fig. 27. The distance from the drop wall to the beginning of hydraulic jump in the alterable pressure

The hydraulic jump at the non-ventilative state was indicated in Fig. 28~30 and by the following equation.

$$\left.\begin{aligned} L_J' &= 2.946 \cdot h_2 \cdot D_n^{-0.084} \\ L_E' &= 3.790 \cdot h_2 \cdot D_n^{-0.0598} \\ L_B' &= 1.027 \cdot h_2 \cdot D_n^{1.55} \end{aligned}\right\} \cdots\cdots\cdots\cdots\cdots\cdots \quad (18)$$

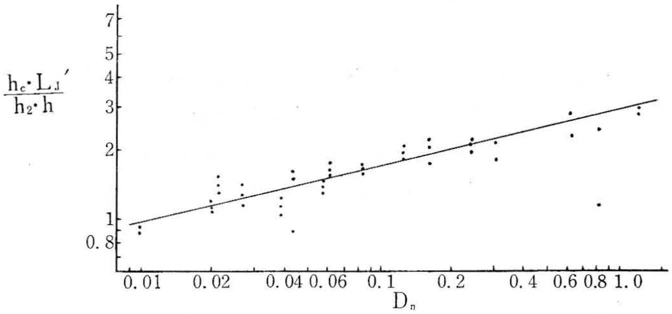

Fig. 28. The length of hydraulic jump at the non-ventilation

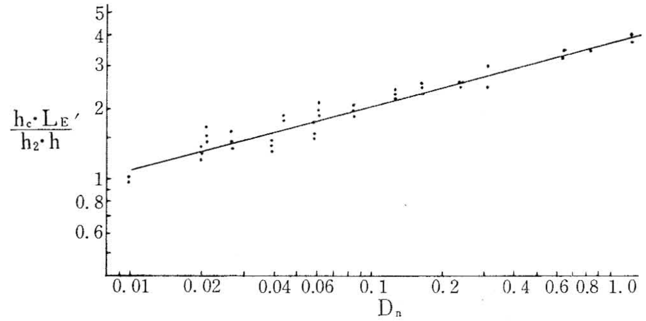

Fig. 29. The distance from the drop wall to the end of hydraulic jump at the non-ventilation

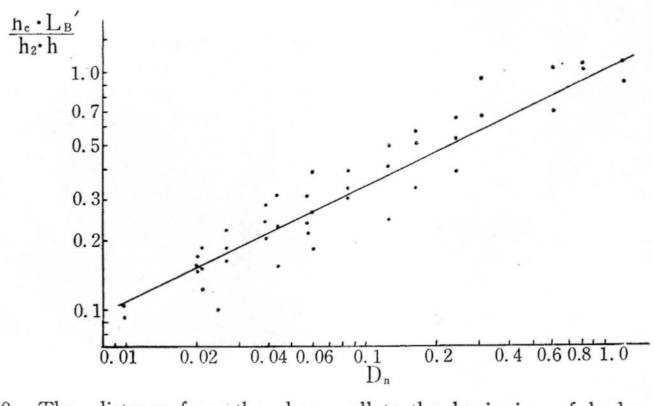

Fig. 30. The distance from the drop wall to the beginning of hydraulic jump at the non-ventilation

The relation between the drop number and $(L_E - L_E')/L_E$ was indicated in Fig. 31. The distance from the crest of the drop to the end of the hydraulic jump decreases approximately thirteen percent at the greatest, in experimental limits.

IV The State in the Downstream Channel

The position of the measurement in the downstream channel was performed at that of two meters from the crest of the drop. Specific energy, distribution of the velocity and the wave flow with the alterable pressure of the back of the nappe did not vary markedly. A meter of the wave height is indicated in Fig. 32.

V Wave Flow

The wave flow occurs when the depth at the crest of the drop is close to the critical depth, it shows the stationary wave in the downstream and switches over to the downstream at the decrement wave gradually. In the complete submerged state, that is, the downstream elevation is high, when the downstream elevation is lowered down step

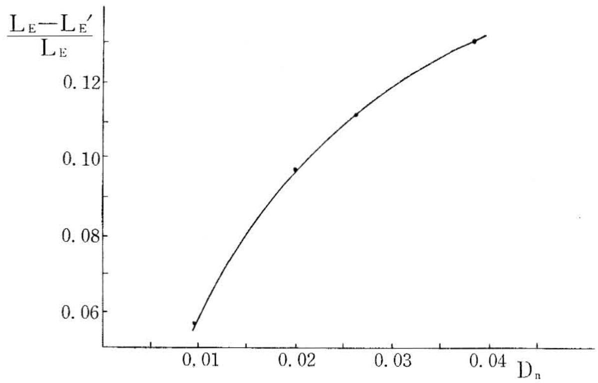

Fig. 31. Proportion of decrease in comparison with the atmospheric pressure and the non-ventilation

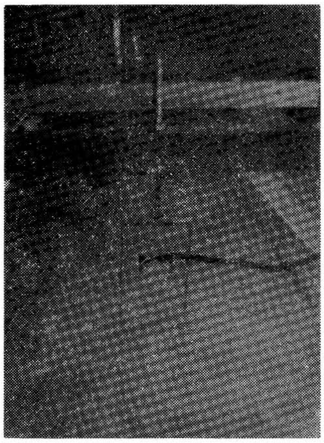

Fig. 32. Meter of wave height

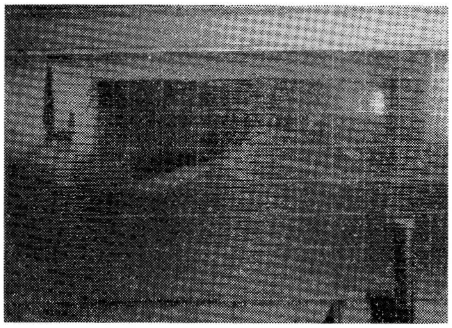

Fig. 33. Wave flow (h = 7 cm, Q = 31.743 e/s)

by step, the wave shape makes amplitude of the wave larger gradually and finally the wave shape breaks. The end of the wave flow differs from that in the case of lifting up the downstream elevation gradually and lowering down the downstream from the previous submerged state. The latter was selected in this paper as the wave shape was kept in the smaller downstream elevation than the former.

An example of the wave flow was indicated in Fig. 33. The beginning and the end of wave flow, making use of the drop number and the Froude number, were indicated in Fig. 34, 35. The relation was indicated by the following equation by the statistical computation.

beginning of wave flow

$$D_n = 1.368 \cdot 10^{-3} \cdot e^{9.567 \cdot \frac{h_0}{h_2}} \qquad (19)$$

end of wave flow

$$D_n = 9.572 \cdot 10^{-4} \cdot e^{16.82 \cdot \frac{h_0}{h_2}} \qquad (20)$$

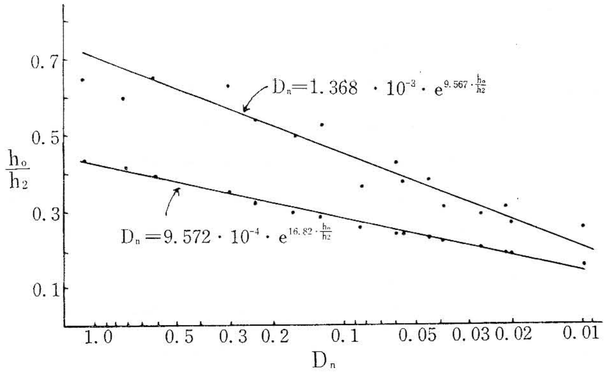

Fig. 34. Limit which wave flow occurs

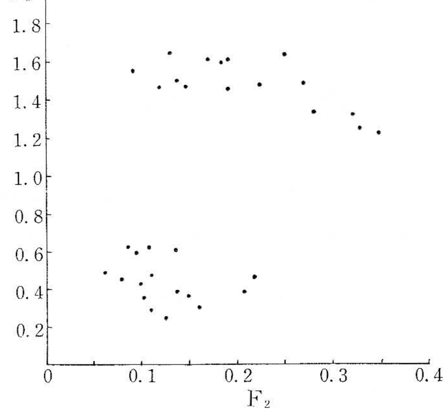

Fig. 35. Indication of limit which wave flow occurs by making use of Froude number

The wave flow occurs within the limit between the beginning and end of the wave flow in Fig. 34, 35. Accordingly, considering the depth in the downstream, the height of the drop and the depth at the crest of the drop in order to avoid this limit, the drop structure must be constructed properly.

Summary

The studies of the flow at the straight drop were carried on in the state in the alterable pressure of the nappe-back or the non-ventilation, because of the decreases of the length of the stilling basin. The result is as follows.

1) In cases the back of the nappe is ventilative and non-ventilative, the writer clarified the relation among the depth at the crest of the drop, the critical depth and a position where arises the critical depth.

2) He serched for the experimental equation in case of the positive pressure, negative pressure and the atmospheric pressure, for a locus of the nappe.

3) As coefficient of the discharge, the angle of inrush and the horizontal distance, which the napp reaches, change with the alterable pressure of the back of the nappe, he showed the proportion of the change by the equation.

4) The wave flow is a large factor for the erosion of the channel and brings about difficulty because of determining the height of the side wall. Accordingly this situation of the flow must be avoided absolutely. He validated that the back of the nappe occurs the non-ventilative state when the downstream elevation becomes that value, and indicated experimentally a danger zone among the depth at the crest of the drop, the critical depth and the height of the drop.

5) As for the distance from the drop wall to the end of the hydraulic jump, he compared the cases of the negative pressure, the atmospheric pressure and the non-ventilation, and showed that the length of the stilling basin can be shortened.

That the negative pressure keeps the back of the nappe, will retain many problems on the dynamics of the construction and the hydraulic, but it is a new trial in the sense that the length of the stilling basin is decreased.

Acknowledgements

The writer wishes to thank Prof. T. KATAOKA, Mr, S. AOKI of IWATE university and Dr. H. SIMURA (Assistant Prof. of Yamagata University) from their load and advice to the work, he wishes to thank Miss H. KOBAYASHI for a fair copy.

References

1) H. ROUSE : Discharge Characteristics of the free overfall, Civil Engineering. vol 6 (4), paper 257–260 (1936).
2) T. IWASAKI : An Experiment Investigation on Hydraulics of the Free Overfall, JSCE, 38–6 (1953).
3) T. KIKUOKA : Some experiments on energy Dissipator of straight drop structure, TAES. 25–8 (1958).

4) T. NAITO, T. YAMAMOTO, M. HOSONO : Rakusakō no Suirikōzō to sono Kinō ni Tsuite, Faculty of Agriculture, Tokyo University of Education (1966).
5) K. IZUTSU : Studies on the hydraulic characteristics of the drop structure, Tech. Bull. Fac. Agr, Kagawa Univ. 18–1, 70–76 (1966).
6) K. ŌTSUDO : Flussigkitsbewegung in der Nähe einer geknickten Flussohle, Ōtsubo Kikutarō Ronbunshū, 49–145 (1939).
7) K. ASHIDA : Kaisuirodanmenkyukakubu no Suiri to Sono Tekiyō ni Kansuru Kenkyū, Disaster Prevention Research Institute Annuals of Kyoto Univ. 5–A (1962).
8) W. P. WHITE : Discussion on "Energy loss at the base of free overfall" by W. L. Moore, 1361–1365 (1943).
9) W. RAND : Flow geometry at the straight drop spillway, Proc. ASCE, vol. 81, 791 (1955).
10) W. L. MOORE : Energy loss at the base of free overfall, NO 2204, 1343–1360 (1852).

要　　　旨

段落流の減勢に関する基礎的研究

前　川　勝　朗

静水池水叩きの長さ節減を計るため段落水脈裏面における圧力を変化させ，あるいは無気層状態として実験を行つた．

研究の成果を列記すると次の通りである．

1) 段落水脈裏面が通気あるいは無通気の場合，落チ口水深，限界水深，落チ口より上流限界水深が生ずる地点までの距離との関係を明らかにした．

2) 段落水脈の軌跡を，段落水脈裏面が正圧，負圧，大気圧の場合について実験的に求めた．流量係数，突込み角，水平到達距離は段落水脈の圧力変化により変りその変化の度合を示した．

3) 波状流は水路にとつて浸蝕の大きな要因であり，また水路高決定に困難をもたらす流れであるのでこのような流水状況は絶対にさけなければならない．段落水脈裏面が無気層となり下流水深がある値になると水路には波状流が生ずることを再確認し，この場合，落チ口水深，限界水深，下流水深，段落高との関係にて危険域を示す実験式を求めた．

4) 落チ口より跳水末端一静水池水叩きの長さについて段落水脈裏面の圧力変化によるその長さの短縮の度合を示し，また大気圧と無気層の場合を比較し短縮の度合を示した．

段落水脈裏面を負圧にすることは，水理学上また構造力学上問題を残すが，実験での負圧の程度は許容範囲内のものであり，その点から静水池水叩きの長さを減ずる新しい試行である．